Non-Gaussianity and Scale Dependence

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"Phenomenological" non-Gaussianity

- assume local non-Gaussianity
- $\zeta(x,t) = f(\phi^{l}_{Gauss}(x,t))$
- $\langle \phi^{l}_{Gauss}(k)\, \phi^{l}_{Gauss}(-k) \rangle \propto H^2 / 2k^3 = G_0(k)$
- assume two Gaussian fields
  - ϕ = inflaton
  - χ = new scalar
- point
  - dominant dependence of ζ on ϕ is linear
  - but dominant non-linearity of ζ depends on χ, not ϕ
  - allows non-Gaussianity consistent with slow-rolling inflaton
- "phenomenological" expansion

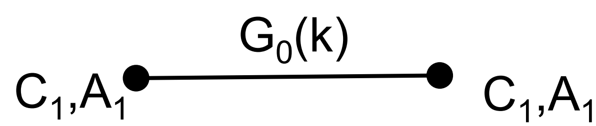
$$\zeta(x,t) = C_1\phi + A_1\chi + \frac{1}{2}A_2\left(\chi^2 - \langle\chi^2\rangle\right) + \frac{1}{6}A_3\chi^3 + \cdots$$

$$\zeta_k = C_1\phi_k + A_1\chi_k + \frac{1}{2}A_2\int\frac{d^3k'}{(2\pi)^3}\chi_{k'}\chi_{k-k'} + \cdots$$

can read local momentum shape from diagrams....

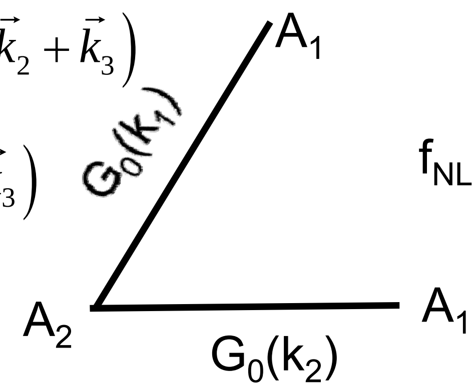
$$\left\langle \zeta\left(\vec{k}_1\right)\zeta\left(\vec{k}_2\right)\right\rangle = (2\pi)^3\left(C_1^2 + A_1^2\right)G_0\left(\vec{k}_1\right)\delta^3\left(\vec{k}_1+\vec{k}_2\right)$$

$$= (2\pi)^3 N^2 H^2 \left(\frac{1}{2k_1^3}\right)\delta^3\left(\vec{k}_1+\vec{k}_2\right)$$

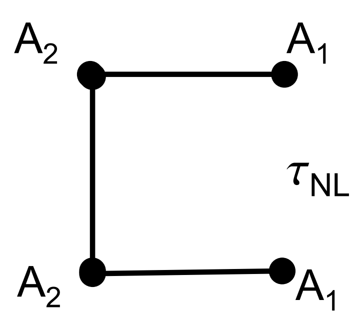


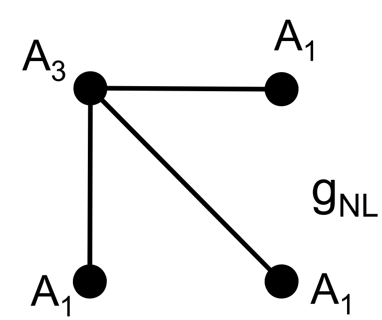
$$\left\langle \zeta\left(\vec{k}_1\right)\zeta\left(\vec{k}_2\right)\zeta\left(\vec{k}_3\right)\right\rangle = (2\pi)^3\left(A_1^2 A_2\right)\left[G_0\left(\vec{k}_1\right)G_0\left(\vec{k}_2\right) + perms.\right]\delta^3\left(\vec{k}_1+\vec{k}_2+\vec{k}_3\right)$$

$$= (2\pi)^3\left(A_1^2 A_2\right)H^4\left[\frac{1}{4k_1^3k_2^3} + perms.\right]\delta^3\left(\vec{k}_1+\vec{k}_2+\vec{k}_3\right)$$



- momentum shape = local
  - dependence of H_{HC}, C_1, A_i on k → scale dependence
  - dH_{HC}/dk , $dC_1/dk \longrightarrow \epsilon,\eta$
  - if A_i const → scale dep. goes as slow-roll parameters





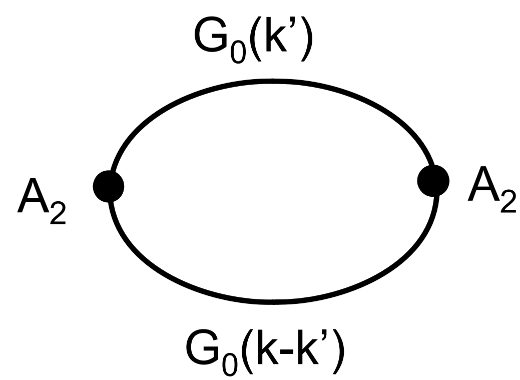
WMAP, Planck, and beyond...

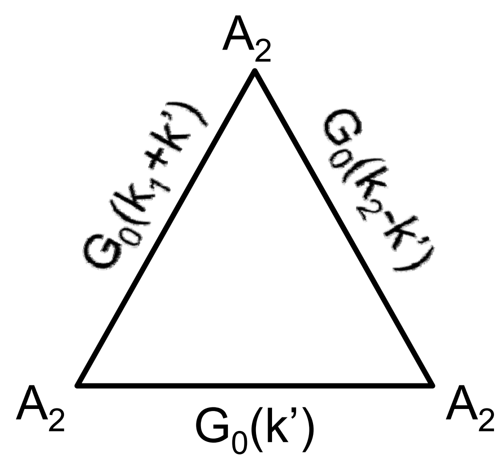
- can probe this soon...
- COBE normalizes 2pt.
- WMAP
  - bounds 2-pt. running
  - bounds on 3-pt. consistent with Gaussian perturbations
  - bounds on 4-pt. could be improved....
- Planck satellite will significantly constrain all of these
- SDSS,Euclid, LSST (larger k)?
- what does local form tell us about NG? Vice versa?

- WMAP
  - $-10 < f_{NL} < 74$ (7 year data)
  - $|\tau_{NL}| < 10^4$ (5 year data)
  - $|g_{NL}| < 10^6$ (5 year data)
- Planck
  - $\Delta f_{NL} < 7$
  - $|\tau_{NL}| < 10^3$
  - $|g_{NL}| < 10^5$ (SDSS comparable)
  - $n_{f_{NL}} \approx 0.1$
- Euclid
  - $|g_{NL}| < 10^4$
- CMBPol,LSST,PanSTARRS, etc. → comparable

scale dependence from loops

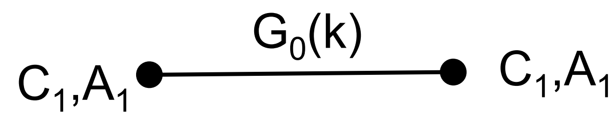
- higher order non-linearities introduce momentum integrals which are not fixed by momentum conservation
- "loop" diagrams
  - induce scale-dependence even if coeff. are constant
  - start with only quadratic terms
- leading scale-dependence in the IR logarithmic
  - $\propto \int d^3k\, k^{-3}$ as loop propagator goes on-shell
- logarithmic IR divergence
- impose IR cutoff



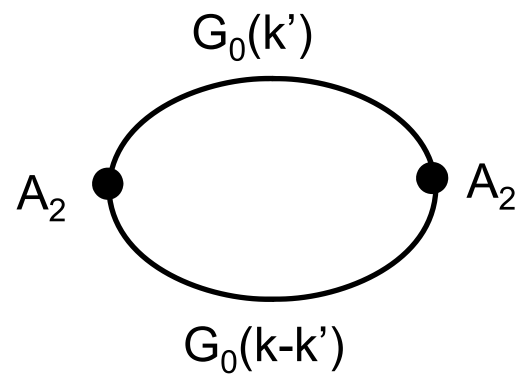


2pt. correlator -- ⟨ $\zeta(k)\,\zeta(-k)$ ⟩

- linear term $\propto N^2 G_0(k)$
- nonlinear term -- χ coupling only



$$A_2^2 \int \frac{d^3k'}{(2\pi)^3} G_0(\vec{k}')\,G_0\left(\vec{k}-\vec{k}'\right)$$



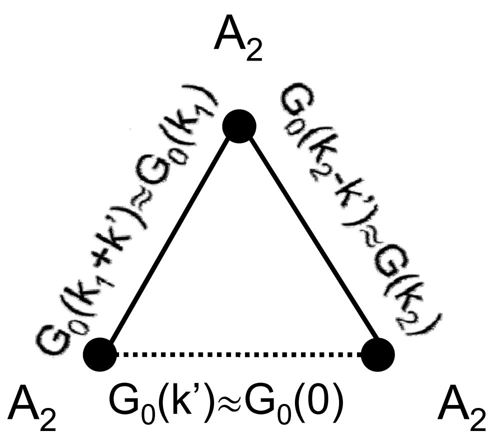
- leading loop contribution from $k' \approx 0,k$
- cut off by denom. when |k'|~|k|
- same shape up to log

$$\sim A_2^2 G_0(k) \int_{k_{IR}}^{k} \frac{d^3k'}{(2\pi)^3} \frac{H^2}{2k'^3} \sim A_2^2 G_0(k) P \ln\frac{k}{k_{IR}}$$

$$P = \left(\frac{H}{2\pi}\right)^2$$

momentum shape....

- we can now see roughly what is happening
  - leading loop integral behavior → one correlator with small momentum inside integral, while other correlators factor outside integral
  - like a tree diagram, with a log factor from momentum integral k_{IR} to k
- wavelengths longer than universe (L) contribute to the "effective" zero-mode variance, and should be treated a constant
  - for $k' < L^{-1}$, mode is treated as a constant and absorbed into a lower-order term
  - $\zeta_k = C_1\phi_k + A_1\chi_k + (1/2)A_2\int (d^3k'/(2\pi)^3)\, \chi_{k'}\chi_{k-k'}$ +... where $k,k' > L^{-1}$
  - swap L^{-1} for k_{IR}



loop correction

- “loop diagram” = “tree-diagram” × F_1
- $F_1 = (A_2/A_1)^2\, P \ln(kL)$ = loop factor
  - (A_2/A_1) factor accounts for different coefficient of the quadratic term
  - P factor accounts for normalization of removed correlator
  - integral over modes from L^{-1} to k generate $\ln(kL)$
  - “k” is a momentum scale set by the external momenta, but its precise value depends on the diagram
- loop and tree diagrams have the same shape, up to $\ln(k)$ corrections
  - loop can dominate, even if perturbation theory valid

f_{NL}

- constraints
  - COBE normalization of the curvature 2pt. function
  - WMAP bounds 2pt. running
  - assume loop term is a small contribution to the 2pt.
- loop contribution bounded
  - loop contribution can dominate the 3pt. correlator if $F_1 > 1$
- resolvable at Planck
  - f_{NL} larger at smaller scales
  - LSS?

- $P^{1/2}N \sim 10^{-5}$
- $n_s - 1 = PA_2^2[N^2 + PA_2^2 \ln(kL)]^{-1}$
- $PA_2^2 / N^2 \leq 10^{-2}$

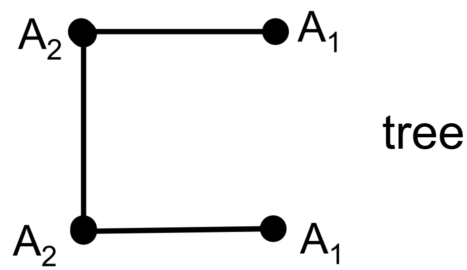
$$f_{NL} \approx -\frac{5}{6}\frac{A_1^2 A_2}{N^4}\left[1 + \frac{A_2^2 P}{A_1^2}\ln(kL)\right]$$

$$\left|f_{NL}^{loop}\right| \approx \frac{5}{6}\frac{\left(PA_2^2/N^2\right)^{3/2}}{P^{1/2}N}\ln(kL) \leq 100\ln(kL)$$

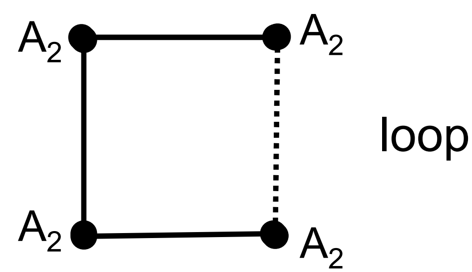
$$n_{f_{NL}} \cong \frac{F_1/\ln(kL)}{1+F_1} \xrightarrow{F_1 \gg 1} \frac{1}{\ln(kL)}$$

quadratic contribution to 4pt

- only generates τ_{NL}
- τ_{NL} controlled by same loop factor (F_1) as f_{NL}



- loop contribution bounded
- if loop term dominates
  - $\tau_{NL} \sim (PA_2^2 / P^{½}N^3)^2 \ln(kL)$
  - $< 10^6 \ln(kL)$

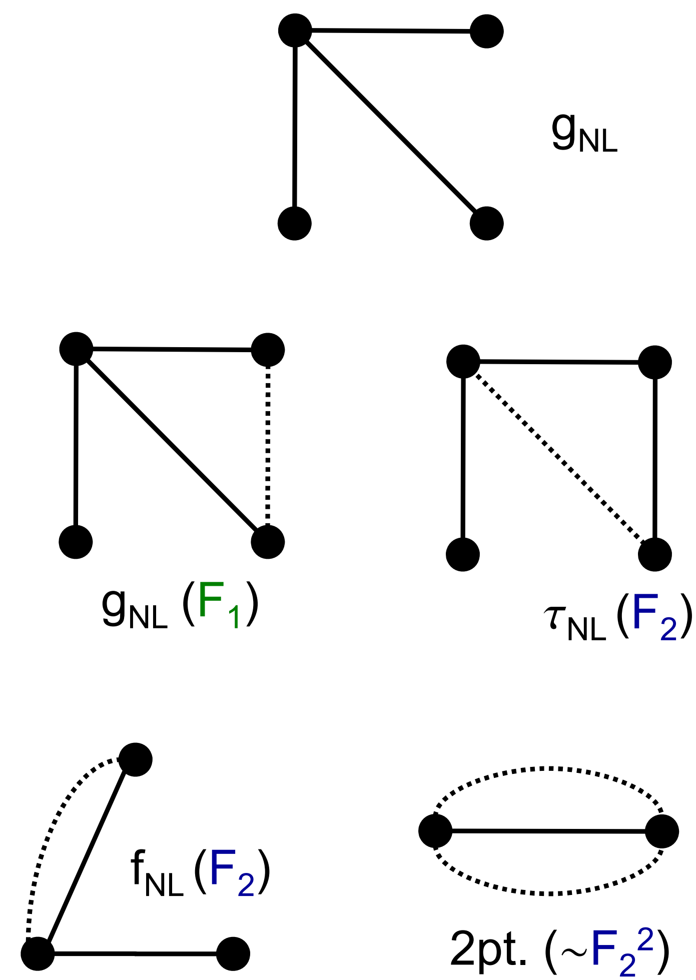


- resolvable at Planck

$$\tau_{NL} \sim \frac{A_1^2 A_2^2}{N^6}\left(1+\frac{A_2^2 P}{A_1^2}\ln(kL)\right) \geq 0$$

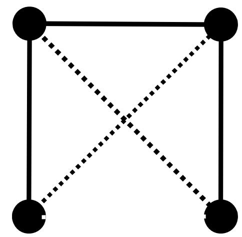
cubic expansion (in progress)....

- new local shape induced for 4pt by cubic interactions
- induces tree contribution to g_{NL}
- 1-loop contribution to f_{NL}, τ_{NL} and g_{NL}
- 2-loop contribution to 2pt.
  - constrained
  - $F_2 = (A_3/A_1)\, P \ln(kL)$
    - new loop factor (<0 ?)
  - if F_2 large, dominant contribution to f_{NL}, τ_{NL} from cubic loop

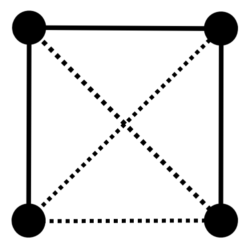


higher loop corrections

- if one-loop contributions can dominate... can higher-loop contributions also? Yes
- but for fixed n-point function and fixed order in non-linearity, loop expansion truncates
- whether or not higher-loop terms needed just depends on ratios A_i / A_j
- also depends on precise dependence of log on external momenta
  - details of which momenta are probed by observations



2-loop correction to τ_{NL}



3-loop correction to τ_{NL}

in progress!

Conclusions

- new data (including Planck) is poised to probe non-Gaussianity in 3pt and 4pt function
- multi-field models can generate non-trivial local spectrum for 3pt and 4pt (both shapes)
- loop diagrams will generate scale dependence even for models where expansion coefficients are constant
- predictions and bounds for loop-dominated models

Mahalo!